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Modeling of an automatic hot water system and heating of a building

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Abstract. A hot water system and heating of a building was studied. The system contains a bio-fuel tank, boiler and radiators. The electro-thermal analogy was used in the modelling of the system and an electrical equivalent scheme was obtained based on the generalized heat model and the identification of the thermal parameters. An automatic water temperature control system with PI controller has been developed. A methodology is proposed for calculating the parameters of the controller under optimum control law and different values of the damping factor. A heat and power consumption in different modes of operation is calculated. Simulation results for the water temperature and the thermal capacity in the leakage of water from the boiler was obtained. The receive results were confirmed experimentally.

1. Introduction

In central heating and domestic water heating systems, boilers of biomass are increasingly used. Biomass boilers have a high efficiency reaching over 90% [1]. Pellets, agricultural product waste, wood chips and hay may be used as biomass [2, 3]. Biomass is clean and renewable energy, the application of which as a source of energy represents a good prospect. Biomass boilers instead of coal boilers can be used in decentralized heating, catering, bathing, swimming pools and other facilities [2].

Two different modeling methods were proposed in study [4]. In the first method, the biomass boiler operates in a steady-state mode at a nominal load of 6 hours. In the second method, the steady-state regime of the system was studied for different heat loads.

In literature [5] a method for adaptive control of the system has been used. The difficulty in developing the model comes from non-linearity, disturbance, inconsistency and the need to take into account the dynamic mode of operation.

The mathematical model in [5] is received from real experiments in the operation of a biomass boiler over a long period of time.

The purpose of this study is to develop a mathematical model of a biomass boiler system. It is necessary to study the obtained model in a dynamic and steady-state mode.

2. Mathematical models

In the development of the mathematical model the electro-thermal analogy was used in which the differences in temperatures, heat flow, thermal resistance and thermal capacitance were replaced

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respectively by potential differences (electrical voltage), electric current, electrical resistance and electric capacitance. In this way, the basic laws of electrical engineering can be recorded for the heating system [3].

A schematic diagram of an existing installation is presented in Figure 1. The biomass boiler has an automatic fueling with a capacity of 40000 kcal/h and a water volume of 80 liters. Throughout the volume, the same water temperature is maintained by a circulation pump (CP). From the boiler output, the heated water enters the radiators and a 300 liter tank connected in parallel (only one of the radiators is shown in the figure). At the input and output of the boiler, the radiators, the tank and the circulation pump, valves are installed. An extension tank (ET) with a capacity of 100 liters is used. A manometer (M) is installed to the boiler to measure the water pressure.

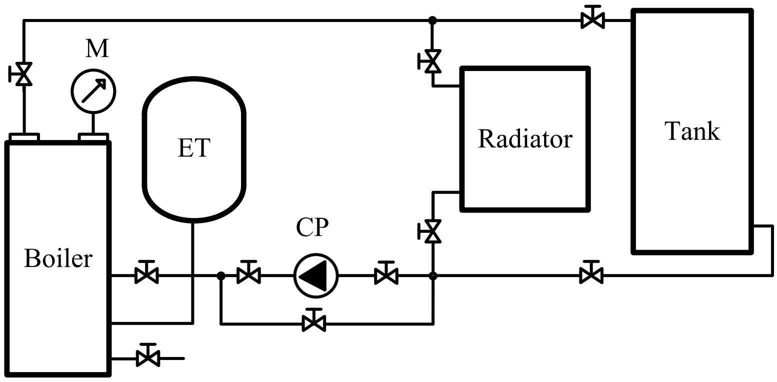

Figure 1. Scheme of the existing installation.

The system can be presented as a common volume with three separate sections - hot water boiler, tank and radiators - figure 2.

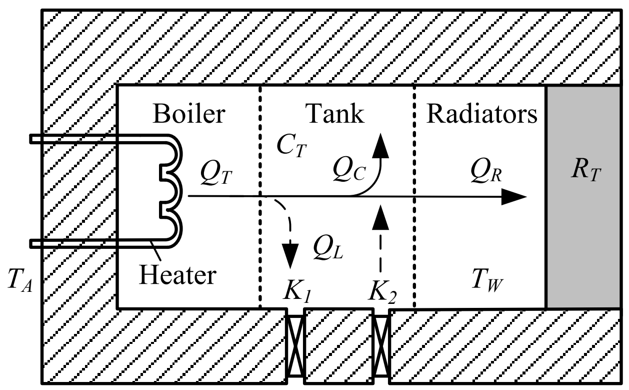

Figure 2. Generalized model of the system.

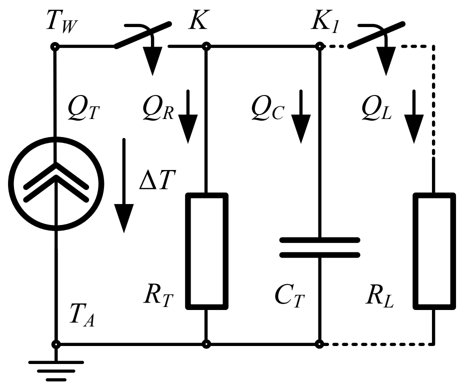

Figure 3. System model represented by electrical circuits.

The volume is covered with an insulating wall that prevents heat dissipation outside. The emitted heat from the radiators in the rooms can be represented by thermal resistance R_T. The input fuel burns at the combustion chamber of pellets (heater), resulting in the heat flow Q_T which heats the boiler, tank and radiators. Part of the heated water leaves the system via the valve K_1. Through the valve K_2 enters cold water to fill the system's volume. Figure 3 shows the electrical equivalent scheme of the heating system. The heat flow Q_T is represented in the electrical equivalent circuit as an ideal current source. The temperature difference of the source terminals is $\Delta T = T_W - T_A$, where T_W is the temperature of the water, and T_A is ambient temperature. The energy (power Q_C) used to heat the water depends on the

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thermal capacity C_T, defined by the expression $C_T = m_W c_P$, where m_W is the mass of water in kg, and c_p is the specific heat of the water in J/(kgK). The energy (power Q_R) that leaves the system in the surrounding space depends on the thermal resistance R_T with the expression $R_T = \Delta T/Q_R$. The energy (power Q_L) that leaves the system when the water leaks from the tank depends on the equivalent leakage resistance R_L, which is determined by the expression $R_L = 1/(G_W c_P)$, where G_W is the mass flow rate in kg/s.

On the basis of the electrothermal analogue for the transition process of a circuit with a capacitor there is obtained:

$$Q_C = C_T \frac{d\Delta T}{dt}. \tag{1}$$

For the node of the electrical equivalent circuit the following equation is recorded:

$$Q_T = Q_R + Q_C = \frac{\Delta T}{R_T} + C_T \frac{d\Delta T}{dt} = \Delta T\left(\frac{1}{R_T} + sC_T\right). \tag{2}$$

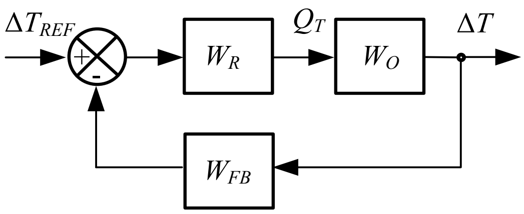

Figure 4. Scheme of closed loop.

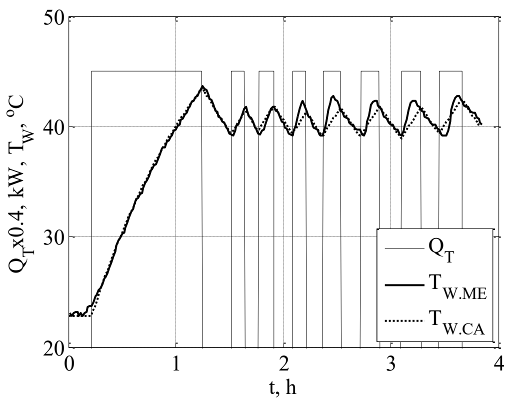

Figure 5. Dependencies Q_T, $T_W = f(t)$.

The transfer function of the system is obtained:

$$W_O(s) = \Delta T / Q_T = 1/(1/R_T + sC_T) = R_T/(1 + sR_T C_T) = k_T/(T_T s + 1), \tag{3}$$

where $k_T = R_T$ is coefficient, and $T_T = R_T C_T$ is the time constant of the system.

In the control system a temperature feedback is introduced by thermocouple with a transfer function $W_{FB}(s) = k_{TC}$. On the basis of the transfer function of the object, a PI controller is selected [6] with a transfer function $W_R(s) = k_s(T_s s + 1)/T_s s$. Figure 4 shows the scheme of the closed loop.

Equivalent transfer function is:

$$W_e = W_R W_O = \frac{k_s(T_s s + 1)}{T_s s}\frac{k_T}{T_T s + 1} = \frac{k_s k_T (T_s s + 1)}{T_s s(T_T s + 1)}. \tag{4}$$

The transfer function of the loop is:

$$W_L(s) = \frac{W_e}{W_{FB} + W_e} = \frac{k_s k_T (T_s s + 1)}{T_s s(T_T s + 1) + k_s k_T (T_s s + 1)} = \frac{k_s k_T T_s s + k_s k_T}{T_s T_T s^2 + (1 + k_s k_T) T_s s + k_s k_T}, \tag{5}$$

where $W_{FB} = 1$. The characteristic equation or the denominator of the transfer function is obtained in a standard form when the two sides of the equation being divided into $k_s k_T$:

$$H(s) = T_s T_T s^2 + (1 + k_s k_T)T_s s + k_s k_T = \frac{T_s T_T}{k_s k_T} s^2 + \frac{(1 + k_s k_T)}{k_s k_T} T_s s + 1 = 0. \quad (6)$$

As a result, the second-order model of the temperature loop is received which has the properties of an oscillating link with a time constant T_0 and damping factor ξ. The characteristic equation is expressed in standard form: $H_C(s) = T_0^2 s^2 + 2\xi T_0 s + 1 = 0$. By equating the closed-loop transfer function to the standard one has been obtained:

$$T_0^2 = \frac{T_s T_T}{k_s k_T}; \quad 2\xi T_0 = \frac{(1 + k_s k_T)}{k_s k_T} T_s. \quad (7)$$

For the damping factor is recorded: $\xi = (1 + k_s k_T)T_s/(4k_s k_T T_s T_T)^{½}$. From the obtained expression you can determine at what value of k_s, ξ there is a minimum through the dependence: $d\xi/k_s = k_s k_T - 1 = 0$. The k_s is obtained: $k_s = 1/k_T$ and for the minimum value of ξ is obtained: $\xi_{min} = (T_s/T_T)^{½}$, where it is expressed: $T_s = \xi_{min}^2 T_T$. The expressions for k_s and T_s are used to calculate the proportional k_p and integral k_i coefficients of PI controller:

$$k_p = k_s = 1/k_T; \quad k_i = k_s / T_s = 1/\left(\xi_{min}^2 k_T T_T\right) \quad (8)$$

The choice of ξ depends on the overshoot and error in the steady-state mode. Several default settings are used in practice: $\xi = 0.999$ - the fastest process without overshoot by the CHR method [7]; $\xi = 0.456$ - the fastest process with overshoot by the CHR method [7]; $\xi = 0.181$ - setting the system of symmetrical optimum [8]; $\xi = 0.707$ - setting the system to a technical (modular) optimum [8].

3. Received results

The developed methodology was experimentally studied through the installation of central heating and domestic water heating. Table 1 summarizes the results of analytical determination of the thermal capacity C_T of the elements in the system.

Table 1. Analytical determination of the thermal capacity of the elements in the system.

System element	m, kg	c_p, kJ/kg.K	C_T, kJ/K
Water in the system	420	4.184	1757.28
Iron vessel on the boiler	150	0.45	67.50
Iron vessel on the tank	150	0.45	67.50
Aluminum radiators	300	0.90	270.00
Total	-	-	2162.28

Figure 5 presents the characteristics of the supply heat power Q_T and и the temperature of the water $T_{W.ME}$ in function of time, measured for 3.83h when the heater is cyclically switched on and off (Q_{Tmax} = 18kW) in reference temperature $T_{W.REF}$ = 45.032°C and ambient temperature T_A = 22.8°C. The average value of the supply thermal power is $Q_{T.AV}$ = 11.2kW. The thermal resistance of the system is obtained by the expression $R_T = \Delta T/Q_{T.AV} = 1.985\times10^{-3}$ K/W, where $\Delta T = T_W - T_A$ = 22.232°C is the difference in temperature. The equivalent thermal capacity C_{Te} is obtained by summing the thermal capacities of the system elements: C_{Te} = 2162.28kJ/K (table 1). Figure 5 also shows the dependence of calculate temperature of water $T_{W.CA}$ in function of time. The figure shows the very good match between measured and calculated results.

The resulting electro-thermal substitution scheme (figure 3) has been simulated during the start of the system (close the K) until the steady state mode is reached, at a constant input heat power Q_T = 11.2 kW. Figure 6 gives the dependencies of Q_T, of the heat flux of the resistor Q_R and the capacitor Q_C in function of time. Figure 7 shows the water temperature dependence T_W as a function of time with a closed-loop with a negative temperature feedback (figure 4) at different ξ (table 2) and a set water temperature ΔT_{REF} = 22.232°C. Table 2 gives the parameters of the PI controller and the error of

T_W relative to the set one at different ξ. The last two columns of table 2 give the error of the maximum value of ΔT relative to the reference one: $\delta_{\Delta Tmax} = (\Delta T_{max} - \Delta T_{REF}).100/\Delta T_{REF}$, % and the error of the steady state value of ΔT relative to the reference one: $\delta_{\Delta Tss} = (\Delta T_{ss} - \Delta T_{REF}).100/\Delta T_{REF}$, %.

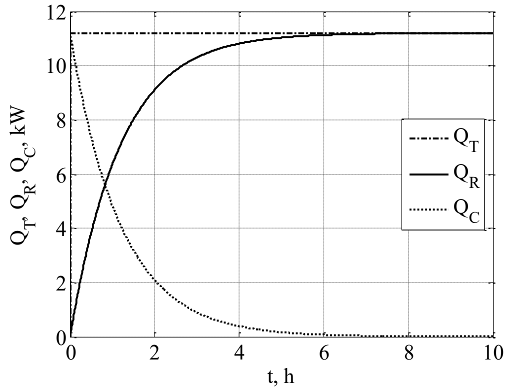

Figure 6. Dependencies Q_T, Q_R, $Q_C = f(t)$.

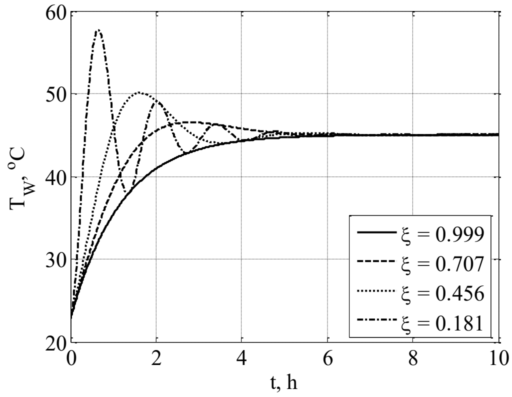

Figure 7. Dependence $T_W = f(t)$ at different values of ξ.

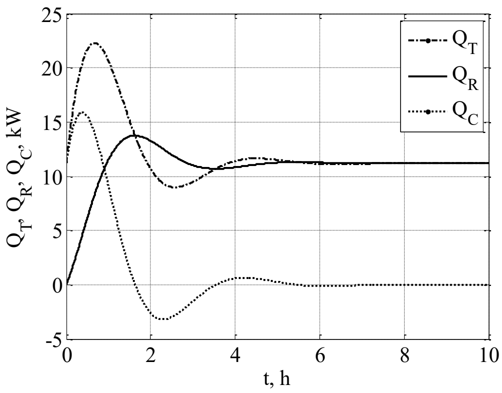

Figure 8. Dependencies Q_T, Q_R, $Q_C = f(t)$.

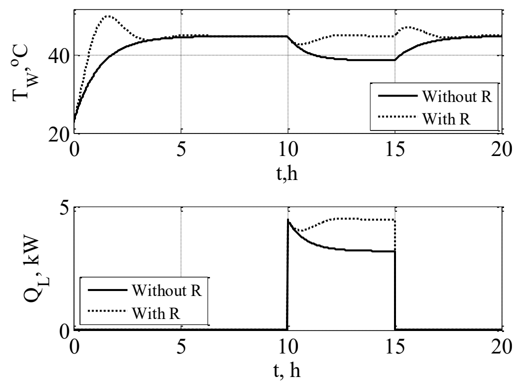

Figure 9. Dependencies $T_W = f(t)$ and $Q_L = f(t)$.

Table 2. PI controller parameters and T_W deviation from the set value at different values of ξ.

ξ	k_p	k_i×10^{-3}	$\delta_{\Delta Tmax}$, %	$\delta_{\Delta Tss}$, %
0.999	503.778	117.608	-0.022	-0.022
0.707	503.778	234.816	6.706	0.011
0.456	503.778	564.465	22.681	0.017
0.181	503.778	3582.695	57.044	0.000

Table 3 presents the results for the average power consumption Q_{AV} by the resistor and the capacitor, as well as the energy W transformed by them into the open and closed system at different ξ. In the same table are given the time to reach the steady state temperature t_{ss} when deviation $\delta = \pm$ 2% [7] and the time to reach the set value t_{set}. The table shows that for the selected value of $\xi = 0.456$ the average power consumption of the resistor and capacitor is approximately equal. The time to reach the set value decreases 5 times, and the energy consumed is increase 10.4 kWh or with 20% versus non-controller mode.

Table 3. Converted powers and energies during the transient process.

ξ	t_{ss}, h	$Q_{R.AV}$, W	W_R, kWh	$Q_{C.AV}$, W	W_C, kWh	W_Σ, kWh	t_{set}, h
Without controller	4.65	8378.1	39.0	2797.9	13.0	52.0	-
0.999	4.63	8373.3	38.8	2831.8	13.1	51.9	4.63
0.707	4.44	9677.4	43.0	5090.7	14.3	57.3	1.77
0.456	4.30	10532.0	45.3	10276.9	17.1	62.4	0.93
0.181	4.30	11077.0	47.6	3071.8	15.4	63.0	0.34

Figure 8 shows the graphs of Q_T, Q_R, Q_C as a function of time with the closed loop and $\xi = 0.456$. The figure shows that the maximum heat input Q_T is 2 times greater than the nominal. The negative Q_C heat flux values correspond to the heat emission from the heated volume through the radiators to the surrounding space.

The electro-thermal equivalent scheme enables to investigate the water leak from the tank by turning on the R_L resistor with key K_1 (figure 3). The mass flow rate is determined experimentally: $G_W = 0.06$ kg/s, from where for the thermal leakage resistance we obtain: $R_L = 3.99\times10^{-3}$ K/W. The flow of water starts at $t_L = 10$h. After time $\Delta t = 5$h, the key K_1 is turned off. Figure 9 shows dependence of the water temperature T_W and the heat flow Q_L released at the water flow as a function of the time for the case without controller (R) and the case with PI controller at $\xi = 0.456$. It can be seen from the graphs that the water temperature drops by 6.3 °C during the leak without a controller and the average thermal power is $Q_{L.AV} = 3395.1$W.

4. Conclusion

A hot water system and heating of a building have been studied. A mathematical model of the system is developed on the basis of the electro-thermal analogy. Thermal power and energy results were obtained in a dynamic and steady state mode. The developed model also allows the thermal parameters to be investigated when leak water from the tank. The received results are compared to experimental to confirm the adequacy of the model.

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