Modeling of an automatic hot water system and heating of a building

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Abstract. A hot water system and heating of a building was studied. The system contains a bio-fuel tank, boiler and radiators. The electro-thermal analogy was used in the modelling of the system and an electrical equivalent scheme was obtained based on the generalized heat model and the identification of the thermal parameters. An automatic water temperature control system with PI controller has been developed. A methodology is proposed for calculating the parameters of the controller under optimum control law and different values of the damping factor. A heat and power consumption in different modes of operation is calculated. Simulation results for the water temperature and the thermal capacity in the leakage of water from the boiler was obtained. The receive results were confirmed experimentally.

1. Introduction
In central heating and domestic water heating systems, boilers of biomass are increasingly used. Biomass boilers have a high efficiency reaching over 90% [1]. Pellets, agricultural product waste, wood chips and hay may be used as biomass [2, 3]. Biomass is clean and renewable energy, the application of which as a source of energy represents a good prospect. Biomass boilers instead of coal boilers can be used in decentralized heating, catering, bathing, swimming pools and other facilities [2].

Two different modeling methods were proposed in study [4]. In the first method, the biomass boiler operates in a steady-state mode at a nominal load of 6 hours. In the second method, the steady-state regime of the system was studied for different heat loads.

In literature [5] a method for adaptive control of the system has been used. The difficulty in developing the model comes from non-linearity, disturbance, inconsistency and the need to take into account the dynamic mode of operation.

The mathematical model in [5] is received from real experiments in the operation of a biomass boiler over a long period of time.

The purpose of this study is to develop a mathematical model of a biomass boiler system. It is necessary to study the obtained model in a dynamic and steady-state mode.

2. Mathematical models
In the development of the mathematical model the electro-thermal analogy was used in which the differences in temperatures, heat flow, thermal resistance and thermal capacitance were replaced
respectively by potential differences (electrical voltage), electric current, electrical resistance and electric capacitance. In this way, the basic laws of electrical engineering can be recorded for the heating system [3].

A schematic diagram of an existing installation is presented in Figure 1. The biomass boiler has an automatic fueling with a capacity of 40000 kcal/h and a water volume of 80 liters. Throughout the volume, the same water temperature is maintained by a circulation pump (CP). From the boiler output, the heated water enters the radiators and a 300 liter tank connected in parallel (only one of the radiators is shown in the figure). At the input and output of the boiler, the radiators, the tank and the circulation pump, valves are installed. An extension tank (ET) with a capacity of 100 liters is used. A manometer (M) is installed to the boiler to measure the water pressure.

![Figure 1. Scheme of the existing installation.](image)

The system can be presented as a common volume with three separate sections - hot water boiler, tank and radiators - figure 2.

![Figure 2. Generalized model of the system.](image)

![Figure 3. System model represented by electrical circuits.](image)

The volume is covered with an insulating wall that prevents heat dissipation outside. The emitted heat from the radiators in the rooms can be represented by thermal resistance $R_T$. The input fuel burns at the combustion chamber of pellets (heater), resulting in the heat flow $Q_T$ which heats the boiler, tank and radiators. Part of the heated water leaves the system via the valve $K_1$. Through the valve $K_2$ enters cold water to fill the system's volume. Figure 3 shows the electrical equivalent scheme of the heating system. The heat flow $Q_T$ is represented in the electrical equivalent circuit as an ideal current source. The temperature difference of the source terminals is $\Delta T = T_W - T_A$, where $T_W$ is the temperature of the water, and $T_A$ is ambient temperature. The energy (power $Q_C$) used to heat the water depends on the
thermal capacity \( C_T \) defined by the expression \( C_T = m_W c_p \), where \( m_W \) is the mass of water in kg, and \( c_p \) is the specific heat of the water in J/(kgK). The energy (power \( Q_R \)) that leaves the system in the surrounding space depends on the thermal resistance \( R_T \) with the expression \( R_T = \Delta T / Q_R \). The energy (power \( Q_L \)) that leaves the system when the water leaks from the tank depends on the equivalent leakage resistance \( R_L \), which is determined by the expression \( R_L = 1 / (G_W c_p) \), where \( G_W \) is the mass flow rate in kg/s.

On the basis of the electrothermal analogue for the transition process of a circuit with a capacitor there is obtained:

\[
Q_C = C_T \frac{d\Delta T}{dt}.
\]

(1)

For the node of the electrical equivalent circuit the following equation is recorded:

\[
Q_T = Q_R + Q_C = \frac{\Delta T}{R_T} + C_T \frac{d\Delta T}{dt} = \Delta T \left( \frac{1}{R_T} + sC_T \right).
\]

(2)

The transfer function of the system is obtained:

\[
W(s) = Q_T / Q_R = 1 / (1/R_T + sC_T) = R_T / (1 + sR_T C_T) = k_T / (T_T s + 1),
\]

(3)

where \( k_T = R_T \) is coefficient, and \( T_T = R_T C_T \) is the time constant of the system.

In the control system a temperature feedback is introduced by thermocouple with a transfer function \( W_{FB}(s) = k_T C \). On the basis of the transfer function of the object, a PI controller is selected [6] with a transfer function \( W_R(s) = k_s (T_T s + 1) / T_T s \). Figure 4 shows the scheme of the closed loop.

Figure 4. Scheme of closed loop.

Figure 5. Dependencies \( Q_T, T_W = f(t) \).

The transfer function of the loop is:

\[
W_L(s) = \frac{W_C}{W_F + W_C} = \frac{k_s k_T (T_T s + 1)}{T_T s (T_T s + 1)} = \frac{k_s k_T (T_T s + 1)}{T_T s^2 + 1 + k_s k_T T_T s + k_s k_T},
\]

(5)

where \( W_{FB} = 1 \). The characteristic equation or the denominator of the transfer function is obtained in a standard form when the two sides of the equation being divided into \( k_s k_T \):
As a result, the second-order model of the temperature loop is received which has the properties of an oscillating link with a time constant $T_0$ and damping factor $\xi$. The characteristic equation is expressed in standard form: $H_c(s) = T_0^2s^2 + 2\xi T_0 s + 1 = 0$. By equating the closed-loop transfer function to the standard one has been obtained:

$$T_0^2 = \frac{T_sT_T}{k_ik_T}; \quad 2\xi T_0 = \frac{(1 + k_sk_T)}{k_ik_T}T_s \quad (7)$$

For the damping factor is recorded: $\xi = (1 + k_sk_T)/T_s/(4k_s^2k_T^2T_T)^{1/2}$. From the obtained expression you can determine at what value of $k_s$, $\xi$ there is a minimum through the dependence: $d\xi/dk_s = k_s - 1 = 0$. The $k_s$ is obtained: $k_s = 1/k_T$ and for the minimum value of $\xi$ is obtained: $\xi_{min} = (T_s/T_T)^{1/2}$, where it is expressed: $T_s = \xi_{min}^2T_T$. The expressions for $k_s$ and $T_s$ are used to calculate the proportional $k_p$ and integral $k_i$ coefficients of PI controller:

$$k_p = k_s = 1/k_T; \quad k_i = k_s / T_s = 1/(\xi_{min}^2k_TT_T) \quad (8)$$

The choice of $\xi$ depends on the overshoot and error in the steady-state mode. Several default settings are used in practice: $\xi = 0.999$ - the fastest process without overshoot by the CHR method [7]; $\xi = 0.456$ - the fastest process with overshoot by the CHR method [7]; $\xi = 0.181$ - setting the system of symmetrical optimum [8]; $\xi = 0.707$ - setting the system to a technical (modular) optimum [8].

### 3. Received results

The developed methodology was experimentally studied through the installation of central heating and domestic water heating. Table 1 summarizes the results of analytical determination of the thermal capacity $C_T$ of the elements in the system.

<table>
<thead>
<tr>
<th>System element</th>
<th>$m$, kg</th>
<th>$c_p$, kJ/kg.K</th>
<th>$C_T$, kJ/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water in the system</td>
<td>420</td>
<td>4.184</td>
<td>1757.28</td>
</tr>
<tr>
<td>Iron vessel on the boiler</td>
<td>150</td>
<td>0.45</td>
<td>67.50</td>
</tr>
<tr>
<td>Iron vessel on the tank</td>
<td>150</td>
<td>0.45</td>
<td>67.50</td>
</tr>
<tr>
<td>Aluminum radiators</td>
<td>300</td>
<td>0.90</td>
<td>270.00</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>-</td>
<td>2162.28</td>
</tr>
</tbody>
</table>

Figure 5 presents the characteristics of the supply heat power $Q_T$ and the temperature of the water $T_{W,ME}$ in function of time, measured for 3.83h when the heater is cyclically switched on and off ($Q_{max}$ = 18kW) in reference temperature $T_{W,REF}$ = 45.032°C and ambient temperature $T_A$ = 22.8°C. The average value of the supply thermal power is $Q_{T,AV}$ = 11.2kW. The thermal resistance of the system is obtained by the expression $R_T = \Delta T/Q_{T,AV} = 1.985 \times 10^3$ K/W, where $\Delta T = T_A - T_w = 22.232^\circ C$ is the difference in temperature. The equivalent thermal capacity $C_{Te}$ is obtained by summing the thermal capacities of the system elements: $C_{Te} = 2162.28$kJ/K (table 1). Figure 5 also shows the dependence of calculate temperature of water $T_{W,CA}$ in function of time. The figure shows the very good match between measured and calculated results.

The resulting electro-thermal substitution scheme (figure 3) has been simulated during the start of the system (close the $K$) until the steady state mode is reached, at a constant input heat power $Q_T = 11.2$ kW. Figure 6 gives the dependencies of $Q_T$, of the heat flux of the resistor $Q_R$ and the capacitor $Q_C$ in function of time. Figure 7 shows the water temperature dependence $T_w$ as a function of time with a closed-loop with a negative temperature feedback (figure 4) at different $\xi$ (table 2) and a set water temperature $\Delta T_{REF} = 22.232^\circ C$. Table 2 gives the parameters of the PI controller and the error of
$T_W$ relative to the set one at different $\xi$. The last two columns of table 2 give the error of the maximum value of $\Delta T$ relative to the reference one: $\delta_{\Delta T_{\max}} = \frac{(\Delta T_{\max} - \Delta T_{\text{REF}})}{\Delta T_{\text{REF}}} \times 100$, % and the error of the steady state value of $\Delta T$ relative to the reference one: $\delta_{\Delta T_{\text{ss}}} = \frac{(\Delta T_{\text{ss}} - \Delta T_{\text{REF}})}{\Delta T_{\text{REF}}} \times 100$, %.

Table 2. PI controller parameters and $T_W$ deviation from the set value at different values of $\xi$.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$k_p$</th>
<th>$k_i \times 10^3$</th>
<th>$\delta_{\Delta T_{\max}}$, %</th>
<th>$\delta_{\Delta T_{\text{ss}}}$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.999</td>
<td>503.778</td>
<td>117.608</td>
<td>-0.022</td>
<td>-0.022</td>
</tr>
<tr>
<td>0.707</td>
<td>503.778</td>
<td>234.816</td>
<td>6.706</td>
<td>0.011</td>
</tr>
<tr>
<td>0.456</td>
<td>503.778</td>
<td>564.465</td>
<td>22.681</td>
<td>0.017</td>
</tr>
<tr>
<td>0.181</td>
<td>503.778</td>
<td>3582.695</td>
<td>57.044</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 3 presents the results for the average power consumption $Q_{AV}$ by the resistor and the capacitor, as well as the energy $W$ transformed by them into the open and closed system at different $\xi$. In the same table are given the time to reach the steady state temperature $t_{ss}$ when deviation $\delta = \pm 2\%$ [7] and the time to reach the set value $t_{set}$. The table shows that for the selected value of $\xi = 0.456$ the average power consumption of the resistor and capacitor is approximately equal. The time to reach the set value decreases 5 times, and the energy consumed is increase 10.4 kWh or with 20% versus non-controller mode.
Table 3. Converted powers and energies during the transient process.

<table>
<thead>
<tr>
<th>ξ</th>
<th>$t_{set}$, h</th>
<th>$Q_{R,AV}$, W</th>
<th>$W_{R}$, kWh</th>
<th>$Q_{C,AV}$, W</th>
<th>$W_{C}$, kWh</th>
<th>$W_{Σ}$, kWh</th>
<th>$t_{set}$, h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without controller</td>
<td>4.65</td>
<td>8378.1</td>
<td>39.0</td>
<td>2797.9</td>
<td>13.0</td>
<td>52.0</td>
<td>-</td>
</tr>
<tr>
<td>0.999</td>
<td>4.63</td>
<td>8373.3</td>
<td>38.8</td>
<td>2831.8</td>
<td>13.1</td>
<td>51.9</td>
<td>4.63</td>
</tr>
<tr>
<td>0.707</td>
<td>4.44</td>
<td>9677.4</td>
<td>43.0</td>
<td>5090.7</td>
<td>14.3</td>
<td>57.3</td>
<td>1.77</td>
</tr>
<tr>
<td>0.456</td>
<td>4.30</td>
<td>10532.0</td>
<td>45.3</td>
<td>10276.9</td>
<td>17.1</td>
<td>62.4</td>
<td>0.93</td>
</tr>
<tr>
<td>0.181</td>
<td>4.30</td>
<td>11077.0</td>
<td>47.6</td>
<td>3071.8</td>
<td>15.4</td>
<td>63.0</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Figure 8 shows the graphs of $Q_T$, $Q_R$, $Q_C$ as a function of time with the closed loop and $\xi = 0.456$. The figure shows that the maximum heat input $Q_T$ is 2 times greater than the nominal. The negative $Q_C$ heat flux values correspond to the heat emission from the heated volume through the radiators to the surrounding space.

The electro-thermal equivalent scheme enables to investigate the water leak from the tank by turning on the $R_L$ resistor with key $K_1$ (figure 3). The mass flow rate is determined experimentally: $G_w = 0.06$ kg/s, from where for the thermal leakage resistance we obtain: $R_L = 3.99 \times 10^{-3}$ K/W. The flow of water starts at $t_L = 10h$. After time $\Delta t = 5h$, the key $K_1$ is turned off. Figure 9 shows dependence of the water temperature $T_w$ and the heat flow $Q_L$ released at the water flow as a function of the time for the case without controller (R) and the case with PI controller at $\xi = 0.456$. It can be seen from the graphs that the water temperature drops by 6.3 °C during the leak without a controller and the average thermal power is $Q_{L,AV} = 3395.1$ W.

4. Conclusion

A hot water system and heating of a building have been studied. A mathematical model of the system is developed on the basis of the electro-thermal analogy. Thermal power and energy results were obtained in a dynamic and steady state mode. The developed model also allows the thermal parameters to be investigated when leak water from the tank. The received results are compared to experimental to confirm the adequacy of the model.

References


