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# **Objects Identification in a Loop with a Two State Controller**

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Abstract. A data collection method is presented, by which identification of high order linear models is achieved. A closed loop experiment with a controller is performed. Frequency characteristics are used and parameters of the model, i.e. the critical operating mode values are obtained. The critical operating mode is realized by two state controller. A system of non-linear equations is compiled, according to the parameters of the model by using the analytical expressions of the amplitude-frequency and phase-frequency characteristics. A method for solving the system is proposed.

### 1. Introduction

The identification of the controllable objects is based on the search for analytical models. They are suitable for easily and repeatedly reproduction of experiments with the object. The results about the properties of the object obtained from the models are with reasonable accuracy. Analytical models can also be used for an experimental setup of the elements of the control device or for producing scaled (reduced or enlarged) physical models [1].

Collecting informative data is a very important stage of the identification, by which an adequate analytical model of the physical processes can be evaluated. The proposed method for obtaining experimental data does not require specialized equipment. For this purpose, a classical linear or state controller is needed. The circuit of the experimental arrangement is the possible wiring diagram for constant operation between the control device and a controllable object, or the so-called circuit for operation in a closed loop [2, 3].

#### 2. Theoretical setup of the identification method

The proposed identification method uses the analytical expressions for: amplitude frequency characteristics (AFC) and phase-frequency characteristics (PFC) for the alleged model and the experimental results for the critical frequency. The analytical expressions for AFC and PFC are known from the control theory (CT) [4] and contain the unknown parameters of the model. The experimental results for the amplitude and the phase are measured from the critical operation mode of the object in the closed loop. Eventually, the task becomes solving a system of non-linear equations, according to the unknown parameters.

figure 1 shows, in general, the circuit of the closed loop with elements and indications of the signals.



Figure 1. General structure and indications in the closed loop

It is indicated in the figure:

CO – Controllable object; CD – Control device (controller); z(t).- Reference value (desired set point); u(t).- Control signal sent to the system; y(t) – The measured output of the system; y(t)- $z(t) = \varepsilon(t)$  – error value.

What is the critical operating mode of the object, and why it is used? The well-known Nyquist stability criterion for stability of linear systems presents best the critical mode. Amplitude-phase-frequency characteristics of the open structure (APFC)  $W_{oc}$  (j $\omega$ ) for three values of the static coefficient of amplification  $K_{p1} < K_{p2} < K_{p3}$ . are presented in figure 2. The coefficient is denoted as  $K_{p1}$ , because it can be considered as the coefficient of the controller.



Figure 2. APFC characteristic of the open structure

According to the Nyquist stability criterion, APFC for  $K_{p1}$  represents a stable closed loop system, and with A3 and  $\Phi$ 3 are marked the corresponding stability margins. APFC for  $K_{p2}$  represents a closed loop system, which is on its limit of stability, and this operating mode is known as critical operating mode. It is known by the theory, that the crossing of the negative part of the real axis of APFC is only possible in case of third and higher order open loop system without transportation lag. The critical operating mode is impossible for first and second order systems.

The parameters of this mode are widely used: for setting the controllers in the loop; as asymptotic quality parameters, which keep performance unchanged; at the design stage and in particular to the identification of the systems.

The critical operating mode is characterized by the following parameters:

Critical frequency  $-\omega_{\kappa p}$ , this is the frequency at which the module of the  $|W_{oc}(j\omega_{\kappa p})| = A(\omega_{\kappa p}) = 1$  and the phase arg $(W_{oc}(j\omega_{\kappa p})) = \Phi(\omega_{\kappa p}) = -\pi$ .

The last of this also implies that the process (for  $K_{p2}$  and  $\omega_{\kappa p}$ ) at the outcome y(t) will oscillate with constant amplitude and frequency, and will be in anti-phase with the input signal u(t). The last fact is used for simple experimental achieving of the critical operation mode, by soft settings variation of the controllers (linear and state).

#### 3. A setting of the closed loop for critical operation mode

The critical operating mode in the closed loop can be realized by two state (on/off) controller.

In case of operation with classic two state controller, the critical operation mode is set as follows - figure 3:



Figure 3. Operation of a two state controller with a high order object

A two state controller is set without non-unique function (hysteresis $\rightarrow$ 0),

$$y_{cp} = y_{3} \tag{1}$$

A setpoint is set z(t) = constant, by absolute value in the middle of the adjustment interval, corresponding to a set point in relative units z% = 50%, where:

$$z\% = \frac{y_{cp} - y_{\min}}{y_{\max} - y_{\min}} 100\%$$
(2)

The variables in the last formula are:

• steady state value which is achieved by maximum control effect in figure 3

$$y_{\max} = K_{o\delta} * u_{\max} \tag{3}$$

• steady-state value which is achieved by minimum control effect in figure 3

$$y_{\min} = K_{o\delta} * u_{\min} \tag{4}$$

•  $y_{cp}$  - the average value of the steady-state oscillations – figure 3

The following parameters are needed for the identification which is calculated with the experimental diagram of the critical process – figure 3:

• critical frequency

$$\omega_{\kappa p} = \frac{2\pi}{T_{\kappa p}} \tag{5}$$

• fluctuation range of the controllable (output) variable

$$\Delta y_{\kappa p} = y_{\text{max}} - y_{\text{min}} \tag{6}$$

• the average value of the control impact for rectangular pulses

$$\Delta u = u_{\max} - u_{\min} \tag{7}$$

• the variation range of the control impact

$$\Delta u = \frac{4}{\pi} \left( u_{\text{max}} - u_{\text{min}} \right) \tag{8}$$

• the average value of the controllable variable

$$y_{cp} = \frac{y_{\max} - y_{\min}}{2} \tag{9}$$

• the average value of the control impact

$$u_{cp} = \frac{\Delta u}{2} \tag{10}$$

#### 4. An algorithm for the calculation method of the parameters of models

Analytical TF, AFC and PFC of the models for order-n, which are defined by the method, are shown in table 1.

№	TF of the model	System of PFC and AFC	
1.	$\frac{K_{o\delta}}{Ts+1} * e^{-\tau s}$	$\pi = \omega_{sp} \tau + \operatorname{arctg} (\omega_{sp} T) \qquad A_{sp} = \frac{K_{o\delta}}{\sqrt{1 + (\omega_{sp} T)^2}} = \frac{\Delta y_{sp}}{\Delta u}$	
2.	$\frac{K_{o\delta}}{\left(Ts+1\right)^2} * e^{-t}$	$\pi = \omega_{\kappa p} \tau + 2 * \operatorname{arctg} (\omega_{\kappa p} T) \qquad A_{\kappa p} = \frac{K_{o\delta}}{1 + (\omega_{\kappa p} T)^2} = \frac{\Delta y_{\kappa p}}{\Delta u}$	
3.	$\frac{K_{o\delta}}{\left(Ts+1\right)^{n}}$	$\pi = n * \operatorname{arctg} (\omega_{sp} T) \qquad A_{sp} = \frac{K_{ob}}{\left(\sqrt{1 + \left(\omega_{sp} T\right)^{2}}\right)^{n}} = \frac{\Delta y_{sp}}{\Delta u}$	

 Table 1. Models of APFC and PFC

The identification method can be depicted with the following algorithm:

- preparation and realization of the experiment and construction of the process in Fig.3. Reading • from the graph:  $u_{min}$ ,  $u_{max}$ ,  $y_{min}$ ,  $y_{max}$  and  $T_{\kappa p}$ . Calculation of  $u_{cp}$ ,  $y_{cp}$  by (9) and (10). calculation of the static coefficient of the object by the formula:
- •

$$K_{o\delta} = \frac{y_{cp}}{u_{cp}} \tag{11}$$

• calculation of the critical frequency by the formula:

$$\omega_{_{\kappa p}} = \frac{2\pi}{T_{_{\kappa p}}} \tag{12}$$

• calculation of the critical module by the formula:

$$A_{sp} = \frac{\Delta y_{sp}}{\Delta u} \tag{13}$$

- for the models 1 and 2 in Table 1., it is easy T and  $\tau$  to be defined first T from AFC and after that  $\tau$  from PFC
- for the models 3, the system is nonlinear according to *T* and *n*. The second equation can be simplified, i. e.

$$A_{\kappa p} = K_{o\delta} \left( \cos \frac{\pi}{n} \right)^n \tag{14}$$

in this form, it is suitable for nomograms.

Methods for solving non-linear equations systems are known from theory.

- it is cleared that the unknown *n* is number of first-order models without lags in the model and can accept equivalent and positive units only, which are greater than or equal to 3, i. e.  $(n \ge 3)$
- the unknown T is mean as a time constant and thus it can accept positive values only, i. e.  $T \ge 0$ .

The systems in table.1 (for the third kind of models) can be solved by building the graphics of the functions:  $n = f_1(T)$ ,  $n = f_2(T)$  and defining the coordinates of the intersection of the graphics. The co-ordinate on the axis (n) is approximated to the nearest equivalent and positive unit. It is accepted as an order model.

The right kind of functions is:

$$n = \frac{\pi}{\operatorname{arctg} (\omega_{_{NP}}T)} = f_1(T)$$
(15)

$$n = \frac{\lg(\frac{K_{ob}}{A_{p}})}{\lg(\sqrt{1 + (\omega_{p}T)^{2}})} = f_{2}(T)$$
(16)

#### 5. Validation of the method, results, and conclusions

The presented method is validated in the computing environment of MATLAB. Numbers of closed loops are simulated with the controller. The controller control objects with different models, from the presented in table 1. In all cases, satisfactory results are reached (considering carefully the graphics). There are no doubts that the method is correct.

For the approbation of the method, an example of fourth-order model is proposed with simulated data. The reader can verify the results, to convince himself in the merits of the method. The variant for achieving a critical operating mode is shown – by the circuit in figure 1. TF of the controllable object is:

$$W_{oy(s)} = \frac{50}{(10\,s+1)^4} \tag{17}$$

And the desired set point is:

$$z(t) = 25 * 1(t) \tag{18}$$

The following quantities are reported from the graphic of the figure. 4:  $u_{min}$ ,  $u_{max}$ ,  $y_{min}$ ,  $y_{max}$  and  $T_{\kappa p..}$ The quantities of  $u_{cp}$ ,  $y_{cp}$  are calculated by (9) and (10).



Figure. 4. A loop with two state controller, without hysteresis -  $u_{min} = 0$  and  $u_{max} = 1$ 

It is calculated from the reported values in table 2:  $\omega_{\kappa p} = 2\pi/T_{\kappa p}$ ;  $K_{o\delta} = y_{cp}/u_{cp}$ ;  $A_{\kappa p} = \Delta y_{\kappa p}/\Delta u$ 

Table 2.					
$\omega_{\kappa p}$	K <sub>об</sub>	$A_{\kappa p}$			
0.0977	50.00	12.5984			

The system of equations (15) and (16) is solved graphically by the obtained values in Table. 2. As a result, the graphics in Fig. 5 are obtained.



Based on the results which are obtained (the coordinates of the intersections), the following conclusions can be drawn:

From the graphic, the results are:

- $T \approx 10s$  and  $n \approx 4$ , which is acceptable accuracy. The error is basically due to the graphical reading of  $y_{cp}$ .
- the aforementioned algorithm is used for the identification of all the models in Table 1, and the results obtained are always with similar accuracy
- the aforementioned algorithm could be used in a closed loop with a PID controller too. The main difference is in the calculation of  $u_{cp}$ , which may be achieved by the integration of u(t).

#### References

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