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Application Of The Momentum 360 Method For Side Collision Between Vehicles Through Computer Simulation

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Abstract. The aim of the article is to demonstrate the reconstruction of a road traffic accident involving vehicles with identical characteristics using the Momentum 360 method through computer simulation. The reconstruction of the accident is presented in side collisions at angles of 45°, 90°, and 135° respectively. In the case of a collision between vehicles at a 45° angle, higher speeds are observed before and during the impact, as evidenced by the measured lengths of the vehicle tracks left after the collision. As the angle increases, the speeds before and during the impact decrease, which is established by the tracks left by both vehicles after the collision. Consequently, an increase in the kinetic energy for the deformation of the vehicles is observed with an increase in the angle of the side impact.

INTRODUCTION

The law of conservation of momentum arises from the mechanics of collision based on impulse - momentum. The impulse - momentum applies the second law of Newton, and this principle alone is applied to the motion of vehicles involved in road traffic accidents. The second law of Newton (change in momentum is equal to impulse) is expressed by the formula [3]:

$$d\left(m\vec{V}\right) = \vec{F}dt \tag{1}$$

where $\vec{F}$ ис the sum of the external forces acting on the system. The impuls $\vec{F}dt$ is a vector that is considered to be equal to (and therefore collinear with) the change in momentum.

The conservation of momentum, by considering each body as a system and treating each vehicle as a system, applies the second law of Newton (the change in momentum is equal to the impulse) separately to each system (vehicle). This takes the form [5]:

$$m\Delta\vec{V} = \vec{p} \tag{2}$$

where $\vec{p}$ is the impulse.

During the reconstruction of a road traffic accident involving vehicles, Equation (1) is integrated over the duration of the vehicle crush phase. This leads to the calculation of the total impulse experienced during this phase and the calculation of the change in impulse that occurred over the interval as shown in Equation (2). Equation (2), in turn, leads to Equation (3) [4] - the law of conservation of momentum represents a vector that has magnitude and direction. Momentum determines the property of a vehicle to maintain motion at a constant velocity and depends on the mass and velocity of the vehicle. Momentum is the product of the mass of the vehicle by its velocity. During a collision of two bodies, forces are applied to them for a very short period. For objects that maintain their shape, it is equal to the change in momentum.

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During vehicle collisions, extremely large forces develop, which usually result in residual deformation. The energy expended for deformation is impossible to determine; therefore, there is not enough information to determine all other collision parameters [2].

The application of the "Momentum 360" method for the law of conservation of momentum in a system of two automobiles. The method can be applied if the exact direction of motion of the vehicles before the impact is known. The presented method, based on the law of conservation of momentum "Momentum 360", can be used for known trajectories of the centers of mass of the vehicles before and after the impact. It is applicable in all cases of road traffic accidents occurring on a straight road or at an intersection where there are skid marks before the impact. The method can be used to investigate accidents involving right-of-way violations at intersections. The law of conservation of momentum of the system of the two automobiles takes the form [6, 5, 1, 7]:

$$\vec{Q} = m_1\vec{V_1} + m_2\vec{V}_2 = m_1\vec{u_1} + m_2\vec{u}_2 \tag{3}$$

where m_1 and m_2 are the vehicle masses;
- $\overrightarrow{V_{1,2}}$ are the velocity of centers of the mass at the moment of the contact;
- $\overrightarrow{u_{1,2}}$ are the velocity of centers of the mass after the moment of the contact.

After setting up the vector equality along the coordinate axes, namely "x" along the axis of travel and "y" perpendicular to it, an algebraic system of two equations concerning the velocities of the vehicles before and after the impact is obtained. The system takes the form:

$$\cos\alpha_1\, m_1V_1 + \cos\alpha_2\, m_2V_2 = \cos\beta_1\, m_1u_1 + \cos\beta_2\, m_2u_2 \tag{4}$$

$$\sin\alpha_1\, m_1V_1 + \sin\alpha_2\, m_2V_2 = \sin\beta_1\, m_1u_1 + \sin\beta_2\, m_2u_2 \tag{5}$$

the $\alpha_{1,2}$ are the angles measured from the abscissa axis to the velocities of the centers of mass of the vehicles before the impact
- $\beta_{1,2}$ are the angles, measured from the abscissa axis to the velocities of the centers of mass of the vehicles after the impact.

After solving the algebraic system of equations, the sought velocities before the impact are determined by the formulas

$$V_1 = \frac{\sin(\beta_1-\alpha_2)m_1u_1+\sin(\beta_2-\alpha_2)m_2u_2}{\sin(\alpha_1-\alpha_2)m_1} \tag{6}$$

$$V_2 = \frac{\sin(\beta_1-\alpha_1)m_1u_1+\sin(\beta_2-\alpha_1)m_2u_2}{\sin(\alpha_2-\alpha_1)m_2} \tag{7}$$

$$u = \sqrt{2g\sum(\mu_i s_i)} \tag{8}$$

where s_i, $i = 1,2$ the distances traveled by the centers of mass of the vehicles after the impact, m;
- μ_i, $i = 1,2$ The coefficients of friction between the tires and the road during the motion of the vehicles after the impact are assumed 0.8.

The loss of kinetic energy during effective braking before the impact, and the velocity of the vehicle is determined solely by the braking distance, assuming it exists:

$$V_{ki} = 0.5t_H j + \sqrt{2js + V_i^2} \tag{9}$$

where t_H the time it takes for the braking deceleration to increase from zero to maximum is assumed 0.4 s;
- j is the maximum braking deceleration of the vehicle, m/s^2.

The maximum braking deceleration of the vehicle during its effective braking is calculated using the formula:

$$j = \mu g, m/s^2 \tag{10}$$

-V_i, $i = 1,2$ is the residual velocity at the end of the skid marks or the velocity at the moment of impact, m/s;
-s the effective braking distance of the vehicle before the impact, m.

The effective braking distance, taking into account the longitudinal base of the vehicle:

$$s_i = s_{СП} - b \quad (11)$$

where b – The longitudinal base of the vehicle;
-$s_{СП}$- braking distance, m.

According to some authors [5], the method may have a significant error because it assumes that the direction of the velocities of the centers of mass before the impact between the vehicles is known. This practically leads to a large physical error, especially in head-on and rear-end collisions. In these cases, the relatively small angle between the longitudinal axes of the vehicles would result in values close to zero in the denominators of equations for velocities (6) and (7), leading to a significant change in the solution. That is, the method is applicable in all cases of road traffic accidents occurring on a straight road or at an intersection when skid marks are available from both vehicles before the impact. It can be used to investigate accidents involving right-of-way violations at intersections. In cases where the method is practically inapplicable, such as in head-on or rear-end collisions on a straight road when there are no skid marks before the impact. The application of the method can be used in aviation [4].

MATERIAL AND METHODS

The Momentum 360 method is most commonly applied in side collisions at intersections involving right-of-way violations. This means that the method can be used if the exact direction of motion of the centers of mass of the vehicles before and after the impact is known. The direction of the centers of mass is traced from the tire marks left by the vehicles. Therefore, the initial position of each vehicle immediately before the impact and the final position after the impact are established. The direction of the velocities of the centers of mass after the impact is assumed to be along the line connecting the center of mass of the vehicle at the moment of impact and the center of mass after the traffic accident.

In the absence of tire marks, the trajectory of the center of mass vector of each vehicle after the impact is usually assumed to be a straight line. In this regard, the distances traveled by the centers of mass should be assumed to be longer.

The accuracy of the Momentum 360 method is determined by the choice of the average coefficient of resistance during the motion of the vehicles after the impact and the reliability of the input angles of the center of mass vectors before and after the impact. The average coefficient of resistance during the movement of the vehicles depends on the type of road surface, the condition of the tires, the actual movement of each vehicle after the impact, the angular velocity at which the vehicle assumes immediately after the impact, the rotation of each of the wheels depending on the applied braking forces, the selected gear, and other factors.

The software product [8] used to create computer simulations of road traffic accidents involving vehicles performs real-time calculations and allows for the design of the human-vehicle-road-environment system. In other words, the software aids in reconstructing road traffic accidents in real-time after correctly applying the Momentum 360 method and properly capturing data from the traffic accident between the vehicles.

The geometry of the collision, as indicated in Figure 1, will be examined in five separate scenarios, in which the movements of the centers of mass of the vehicles will be established both before and after the impact, with tire marks present before and after the impact, respectively. In other words, the orientations and movements of the vehicles involved in a side collision with different geometries will be determined. For this purpose, the side collision between the vehicles will be reconstructed in five scenarios evenly spaced apart, covering the entire length of the vehicle. These collisions will be reconstructed at three separate angles: 45°, 90°, and 135°, respectively, simulating different types of intersections. For this purpose, two vehicles with identical characteristics, as specified in Table 1, will be used. The working speeds for normal vehicle movement will be assumed to be 90 km/h for both vehicles before anticipating the occurrence of the road traffic accident.

The resistance to motion of the vehicles on a hard road surface is assumed to be constant, i.e., equal to 0.012. This means that the resistance to motion of the vehicle arises due to hysteresis losses from the radial and tangential deformation of the tires resulting from their friction with the road.

The speed of both vehicles is assumed to be 90 km/h up to the time of the drivers' reaction.

TABLE 1. Vehicle characteristics

Make	Model	Production period	Engine displacement, cm^2	Brake Power, kW	Mass of the vehicle, kg	Vehicle width, mm	Track, mm	Vehicle length, mm
BMW	525i	2000 – 2003	2494	141	1580	2830	1510	4780

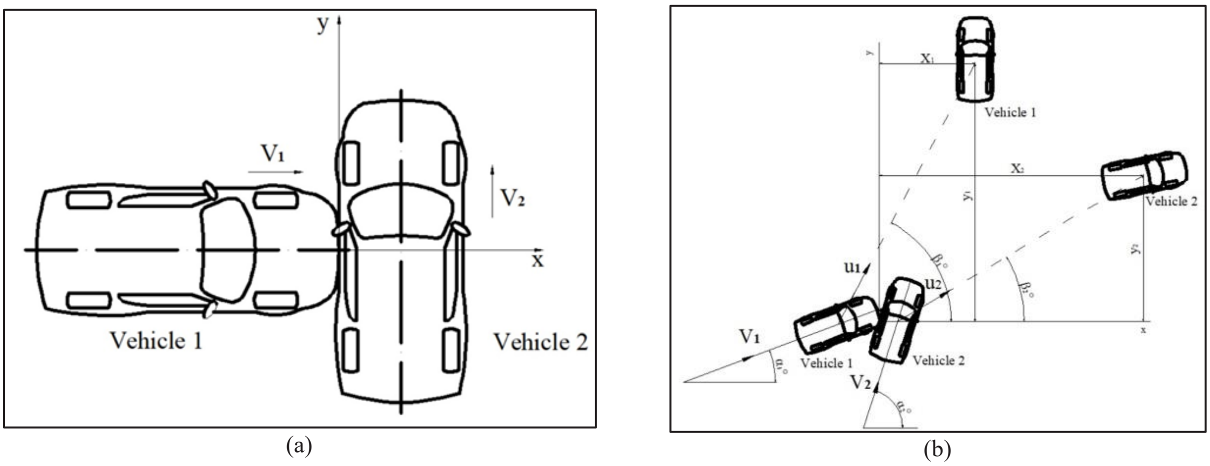


FIGURE 1. Side collision of vehicles:
(a) side collision with the vectors of the vehicles' velocities;
(b) parameters of the side collision between the vehicles.

RESULTS AND DISCUSSION

In many cases, predicting the reactions of drivers involved in a road traffic accident is difficult. Immediately before impact, one of the decisive factors is recognizing the impending accident and the drivers' reaction time. The steps needed to take action to avoid the accident by the vehicle drivers consist of activating the braking systems and the time it takes for the braking devices to engage.

During the investigation of road traffic accidents, a series of factors are taken into consideration, including the type of road surface, determining the coefficient of friction, the technical condition of the vehicles, weather conditions, and not least, the reactions of the drivers and their duration.

The reactions of drivers depend on factors primarily related to accumulated experience in operating motor vehicles, gender, health condition, including hearing, vision, reflexes, normal functioning of the musculoskeletal system, and others. Another factor is the high speed of the vehicles and the short reaction time.

The reconstructed five cases of side collisions executed at three different angles lead to different lengths of brake marks left by the two vehicles respectively. The drivers of the vehicles have anticipated the impending road traffic accident and have taken steps to avoid it. They have applied pressure to the brake pedals, activated the braking systems and devices of the vehicles, and turned the steering wheel to avoid the collision.

Figures 2, 3, and 4 depict the side collisions at angles of 45°, 90°, and 135° respectively, at five equal distances along the entire length of vehicle No. 2. The braking distance remains almost unchanged before the collision at all three angles, as indicated by the measurements of tire marks. From the graphs in Figure 5, it becomes clear that the distances covered in a side collision at a 45° angle are the longest because the kinetic energy separated during this contact is the smallest, and consequently, the speeds after the collision remain the highest. In contrast, collisions at a 135° angle leave the shortest tire marks, indicating the highest kinetic energy during the collision, and consequently, the deformation is expected to have the highest values. Therefore, as the angle of side collision increases, the separated kinetic energy increases, leading to a decrease in speeds before and during the collision.

Reconstructions at the moment the most powerful kinetic energy we all see in side collisions of 135°. The vehicles have the shortest braking distance and the deformation on a given kinetic energy basis is the largest. This side collisions comes closest to a frontal impact especially in Case 1 (Figure 4) where we have an impact the front side of the both vehicles.

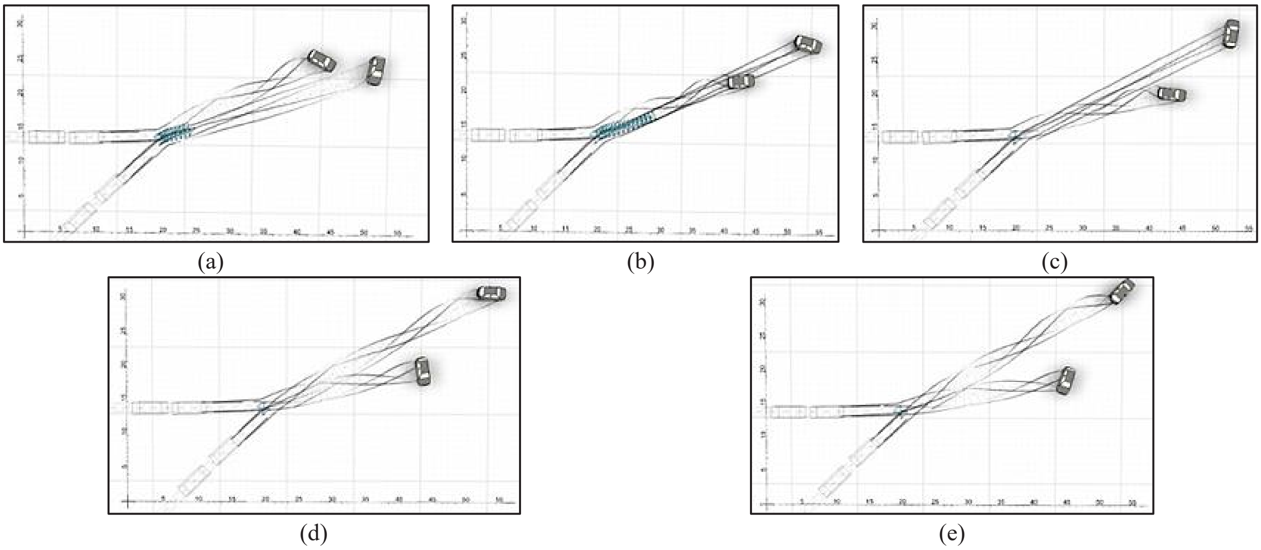


FIGURE 2. Side collision at a 45° angle in five separate scenarios at equal intervals::
(a) case 1; (b) case 2; (c) case 3; (d) case 4; (e) case 5.

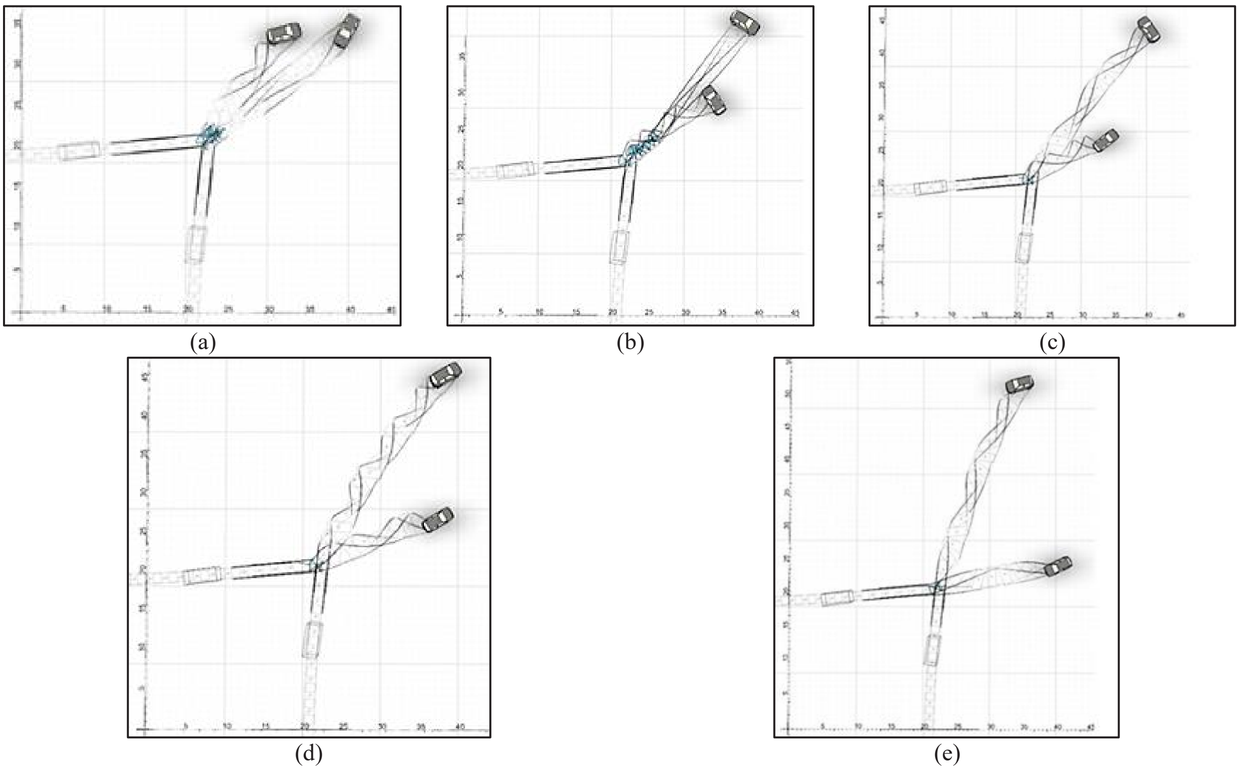


FIGURE 3. Side collision at a 90° angle in five separate scenarios at equal intervals::
(a) case 1; (b) case 2; (c) case 3; (d) case 4; (e) case 5.

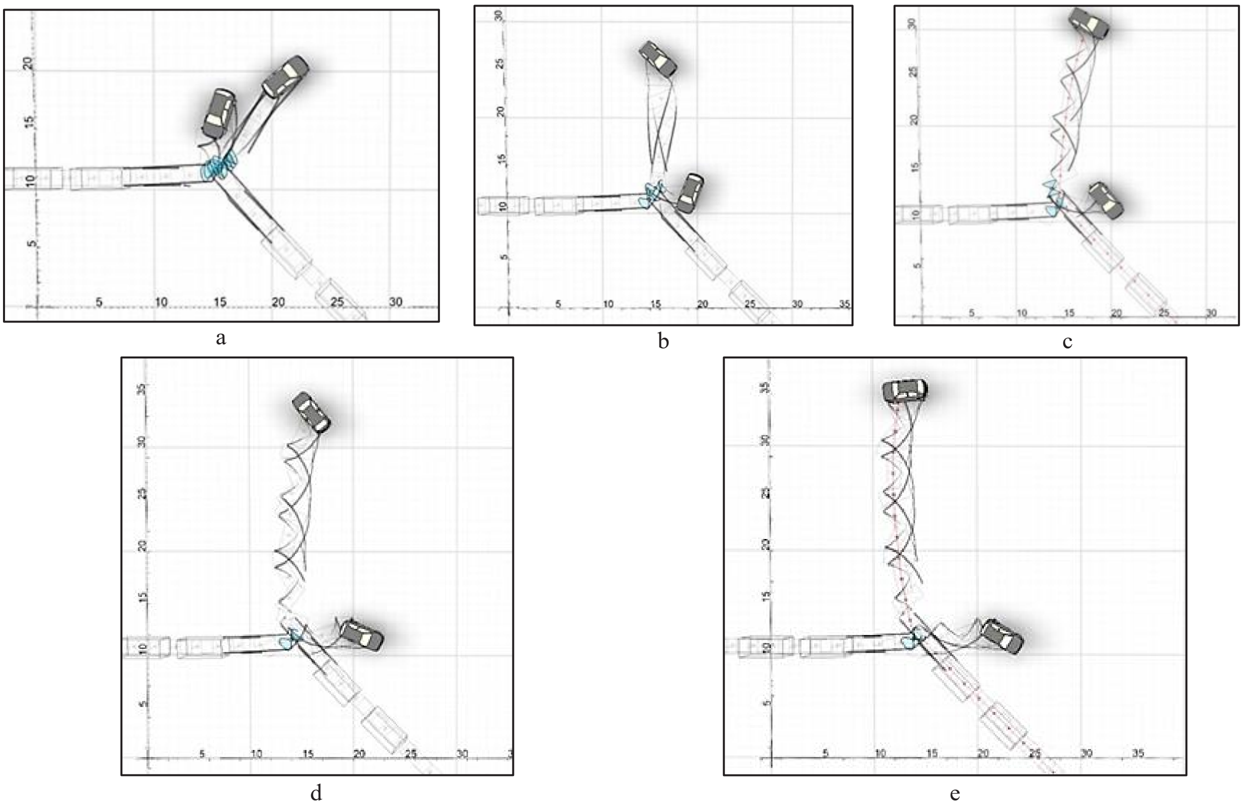
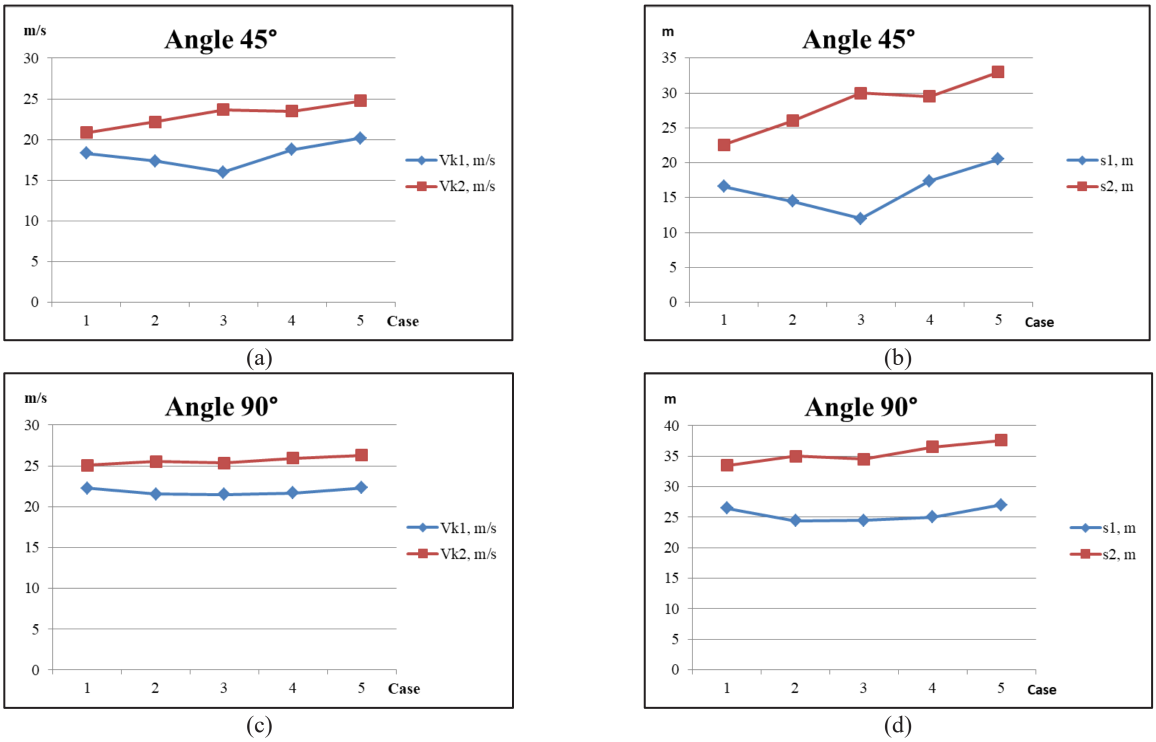
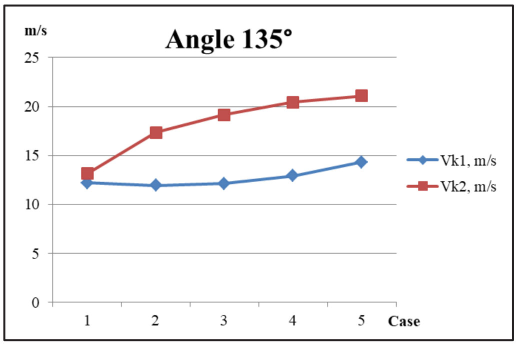
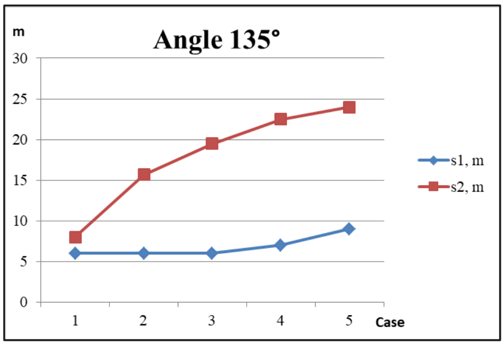


FIGURE 4. Side collision at a 135° angle in five separate scenarios at equal intervals::
(a) case 1; (b) case 2; (c) case 3; (d) case 4; (e) case 5.





(e)



(f)

FIGURE 5. Distances traveled by the centers of mass of the vehicles after the collision and speeds after the loss of kinetic energy due to effective braking before the collision in the respective three angles and five scenarios of road traffic accidents: (a), (c), (e) speeds; (b), (d), (f) distances traveled

CONCLUSION

The skid marks after a collision at a 45° angle, left by the vehicles, are the longest, suggesting that there is less kinetic energy expended for their deformation during the collision. As the angle increases to 135°, there is a decrease in the length of the skid marks after the collision, indicating that more kinetic energy is expended during the collision for the deformation of the vehicles. From all cases considered at the moment the most powerful kinetic energy we all see in side collisions of 135°. The vehicles have the shortest braking distance and the deformation on a given kinetic energy basis is the largest.

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